ONE-DIMENSIONAL PROPAGATION OF A MONOCHROMATIC LIGHT PULSE IN ABSORBING MEDIA

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UDC 535.211

A study is made of the equations describing the propagation of a monochromatic pulse of radiation of arbitrary shape in absorbing media – a plasma and an absorbing two-level photodissociable medium. Exact analytic solutions are found for a wide variety of boundary conditions. The discussion is carried through for problems with plane, cylindrical, and spherical symmetry. The formulas obtained can be used directly to compare calculation and experiment.

The equations for the transport of radiation in media whose properties are altered by a light pulse are nonlinear. This significantly limits the number of cases for which it is possible to obtain a closed analytic solution of the system of equations satisfying prescribed initial and boundary conditions. Particular solutions of the transport equations in an absorbing (amplifying) two-level medium and in a plasma have been obtained by various methods [1-4].

The equation for the transport of radiation of frequency ν in an absorbing (amplifying) medium has the form

$$\frac{1}{c}\frac{\partial I}{\partial t}+\Omega\nabla I=-KI \tag{1}$$

where $I(\mathbf{r}, \Omega, t)$ is the intensity of radiation per unit solid angle propagating in the direction Ω, K is the absorption coefficient of the medium, and c is the velocity of light. We note that the first term in (1) is essential in investigating picosecond pulses in a plasma [4, 5]. When the reflection and scattering of light can be neglected and propagation in the medium can be treated as one-dimensional, Eq. (1) takes the form

$$\frac{1}{c}\frac{\partial I}{\partial t}+\frac{1}{r^{\alpha}}\frac{\partial}{\partial r}(r^{\alpha}I)=-KI \tag{2}$$

where $\alpha=0$, 1, and 2 respectively for plane, cylindrical, and spherical symmetry.

The material equation for an absorbing medium can be written as

$$an(r)\partial\varphi/\partial t=KI \tag{3}$$

by assuming that the medium is characterized by two variable parameters $\varphi(r, t)$, $n(r)$, and $K=K(\varphi, n)$. We write the initial and boundary conditions in the form

$$I(r,0)=0,\quad \varphi(r,0)=\varphi_*(r)\quad (r\geqslant r_0)$$
$$I(r_0,t)=I_0(t)\quad (t\geqslant 0) \tag{4}$$

Equations (2) and (3) with conditions (4) constitute the mathematical formulation of a number of physical problems:

1) If $\varphi=T$ is the plasma temperature, n is the electron density, $a=3/2$, and $K=K(T, n)$, we have the problem of the heating of a quiescent plasma by laser radiation [1, 3, 4];

Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 5, pp. 177-180, September-October, 1972. Original article submitted January 6, 1972.

2) if $\varphi=N$ is the molecular density of the gas, $a=-1$, $n=1$, $K=\sigma N$, where σ is the photolysis cross section, we have the problem of the propagation of a photodissociation wave in a gas [6];

3) if $\varphi=N_2-N_1$ is the difference in populations of the excited and ground states of active atoms, $K=\sigma(N_1-N_2)$, $n=1$, $a=1/2$, we have the problem of the amplification (absorption) of a light pulse in a two-level medium without taking account of the spontaneous decay of the upper level [2, 7].

After the change of variables

$$\eta = ct - r + r_0,\quad \xi = r \tag{5}$$

the system (2), (3) takes the form

$$\frac{\partial i}{\partial \xi} + \frac{\alpha}{\xi} i = -Ki \tag{6}$$

$$n(\xi)\frac{\partial \varphi}{\partial \eta} = Ki \tag{7}$$

Here we have introduced the notation $i=I/ac$. If K is written in the form $K=k(\xi)/f(\varphi)$ and i is eliminated from the equations, we find a first integral of the system (6), (7):

$$\frac{\partial}{\partial \xi}\left[\frac{n}{k}\int f(\varphi)\,d\varphi\right] + \frac{\alpha}{\xi}\frac{n}{k}\int f(\varphi)\,d\varphi + n\varphi = Q_1(\xi) \tag{8}$$

For a wide variety of physical problems we can set [1, 8]

$$f(\varphi) = \varphi^{\gamma},\qquad k = k_0 n^{\beta}$$

For example, for bremsstrahlung-type absorption in a plasma $\gamma=3/2$, $\beta=2$; in photolysis or light amplification in an active medium $\gamma=-1$, $\beta=0$, and $k_0=\sigma$. We investigate Eq. (8) for two cases:

Case 1: $\gamma\neq-1$. Equation (8) reduces to the generalized Bernouilli equation for the function $\Omega = n^{1-\beta}\varphi^{\gamma+1}$:

$$\dot{\Omega} = -A(\xi)\,\Omega^{-(\gamma+1)^{-1}} - \frac{\alpha}{\xi}\Omega + Q(\xi),\quad A(\xi) = (\gamma+1)\,k_0 n^{\varkappa},\quad \varkappa = \frac{\gamma+\beta}{\gamma+1} \tag{9}$$

The solution of (9) is reduced to quadratures if $Q(\xi)$ satisfies the equation

$$\frac{dQ^{\gamma+1}}{d\xi} + \left(\frac{\alpha}{\xi} - \frac{\gamma+1}{A}\frac{dA}{d\xi}\right)Q^{\gamma+1} - R(-A)^{\gamma+1}Q = 0 \tag{10}$$

from which we find that Q must have the form

$$Q(\xi) = (\gamma+1)\,k_0 n^{\varkappa}\xi^{-\chi}\left(G - \gamma\delta k_0 R\int n^{\varkappa}\xi^{\gamma\chi}d\xi\right)^{1/\gamma},\quad \chi = \frac{\alpha}{\gamma+1} \tag{11}$$

where R and G are arbitrary constants, $\delta=(-1)^{\gamma}$ for $\gamma>-1$, and $\delta=1$ for $\gamma<-1$.

For $Q(\xi)$ in the form (11) the solution of (8) has the form

$$\int\frac{d\omega}{\omega^{1/(\gamma+1)} - R\omega + 1} + F(\eta) = \frac{(-1)^{\gamma}(\gamma+1)}{\gamma\delta R}\ln\left|\gamma\delta k_0 R\int n^{\varkappa}\xi^{\gamma\chi}d\xi - G\right| \tag{12}$$

where

$$\omega = [-A(\xi)/Q(\xi)]^{\gamma+1}\,\Omega$$

Using boundary condition (4) at $r=r_0$ we find $F(\eta)$. Finally we have

$$\int\limits_{\omega_0}^{\omega}\frac{d\omega}{\omega^{1/(\gamma+1)} - R\omega + 1} = \frac{(-1)^{\gamma}(\gamma+1)}{\gamma\delta R}\ln\left|\left[\gamma\delta k_0 R\int^{r} n^{\varkappa}r^{\gamma\chi}dr - G\right]\left[\gamma\delta k_0 R\int^{r_0} n^{\varkappa}r^{\gamma\chi}dr - G\right]^{-1}\right|$$

where

$$\omega = (-1)^{\gamma+1}r^{\alpha}n^{1-\beta}\varphi^{\gamma+1},\qquad \omega_0 = (-1)^{\gamma+1}r_0^{\alpha}n^{1-\beta}(r_0)\,\varphi_0^{\gamma+1}$$
$$\varphi_0(\eta) = \left[k_0(\gamma+1)\,n(r_0)\,a^{-1}\int\limits_0^{\eta/c} I_0(\eta)\,d\eta + \varphi_*^{\gamma+1}(r_0)\right]^{1/(\gamma+1)} \tag{13}$$

The expression for the intensity I is easy to obtain from (7):

$$I = I_0(\eta)\,[\omega^{1/(\gamma+1)} - R\omega + 1]\,[\omega_0^{1/(\gamma+1)} - R\omega_0 + 1]^{-1} \tag{14}$$

We use the initial condition (4) at t=0. Since I(r, 0)=0 we have for t=0

$$\partial/\partial\xi \equiv \partial/\partial r, \; Q(\xi) \equiv Q(r)$$

Specifying Q(ξ) in the form (11) is equivalent to specifying the initial profiles of n(r) and φ_*(r); the expressions for these quantities must satisfy the equation

$$\int\limits_{\omega_{*0}}^{\omega_*} \frac{d\omega}{\omega^{1/(\gamma+1)} - R\omega + 1} = \frac{(-1)^\gamma(\gamma+1)}{\gamma\delta R} \ln\left| \left[\gamma\delta k_0 R \int\limits^{r} n^\varkappa r^{\gamma\chi} dr - G\right] \left[\gamma\delta k_0 R \int\limits^{r_0} n^\varkappa r^{\gamma\chi} dr - G\right]^{-1}\right|$$

where

$$\omega_* = (-1)^{\gamma+1} r^\alpha n^{1-\beta} \varphi_*^{\gamma+1}, \qquad \omega_{*0} = \omega_*(r_0) \tag{15}$$

Thus the system (2)-(4) with condition (15) has a solution obtained from (13) and (14). In particular the expressions given in [1, 3, 4] are easily obtained from Eqs. (13) and (14) by setting R=0 and giving n(r) and φ_*(r) special forms. It should be noted that, for example, in problems of plasma heating by intense laser pulses it is important to obtain analytic solutions for the most diverse initial profiles of the plasma density and the temperature for various types of symmetry in order to be able to compare the calculations with experimental data. For example, when picosecond pulses are incident on a solid target [5] the profile of the plasma density n(r) is an increasing function of r as a consequence of the gas dynamic removal of the heated layer, and the profile of the plasma temperature is bell-shaped since the temperature is low at the plasma-vacuum boundary because of gas-dynamic dispersion and at the plasma-solid boundary because of the loss of heat by electron thermal conduction; the motion of the plasma during the time of propagation of the pulse can be neglected because the time involved is so small (10^{-11}-10^{-12} sec). The required form of the profile is selected by varying R and G in Eq. (15).

Case 2: $\gamma=-1$. For $\gamma=-1$ Eq. (8) takes the form

$$\frac{1}{\varphi}\frac{\partial\varphi}{\partial\xi} + \left[\frac{(1-\beta)}{n}\frac{dn}{d\xi} + \frac{\alpha}{\xi}\right]\ln\varphi + k_0 n^\beta \varphi = Q_1(\xi) \tag{16}$$

In contrast with the case considered above it is expedient to solve Eq. (16) separately for each physical case since the search for a general method of solution is beset with formidable mathematical difficulties. In particular let us consider Eq. (16) as applied to photolysis problems or to the propagation of a pulse in a two-level medium. Setting n=1 and $k_0=\sigma$ (the interaction cross-section for the appropriate process) we obtain

$$\frac{1}{\varphi}\frac{\partial\varphi}{\partial\xi} + \frac{\alpha}{\xi}\ln\varphi + \sigma\varphi = Q_1(\xi) \tag{17}$$

If $\alpha=0$ (plane case) Eq. (17) is linearized by the substitution $Z=\varphi^{-1}$, and its solution for a two-level medium ($\varphi=N_2-N_1$) is given in [2]. For $\alpha\neq 0$ the substitution $\omega=\sigma\varphi\xi$ reduces Eq. (17) to the form

$$\frac{\xi}{\omega}\frac{\partial\omega}{\partial\xi} + \alpha\ln\omega + \omega = Q(\xi) \tag{18}$$

For Q=const and the boundary condition (4) the solution of Eq. (18) can be written in the form

$$\int\limits_{\omega_0}^{\omega} \frac{d\omega}{\omega(Q-\omega-\alpha\ln\omega)} = \ln\frac{r}{r_0} \tag{19}$$

where

$$\omega_0 = \sigma r_0\varphi_0, \qquad \varphi_0 = \varphi_*(r_0)\exp\left[-\frac{\sigma}{a}\int\limits_0^{\eta} I_0(\eta)\,d\eta\right]$$

The expression for the intensity can obviously be written in the same form as (14):

$$I = I_0(\eta)\,[\omega + \alpha\ln\omega - Q]\,[\omega_0 + \alpha\ln\omega_0 - Q]^{-1} \tag{20}$$

We now take account of the initial condition at t=0. The profile of $\varphi_* = \omega_*(r)/\sigma r$ must satisfy the equation

$$\int\limits_{\omega_*(r_0)}^{\omega_*} \frac{d\omega}{\omega(Q-\omega-\alpha\ln\omega)} = \ln\frac{r}{r_0} \tag{21}$$

where $Q \neq Q^* = \alpha \ln \omega_*(r_0) + \omega_*(r_0)$. If $Q = Q^*$ the profile of φ_* has the simple form

$$\varphi_* = \varphi_*(r_0)\, r_0/r$$

In cylindrical symmetry this form of the profile is in a certain sense equivalent to a constant profile of φ_* in plane symmetry; i.e., the wave of the corresponding process for a constant intensity I_0 at the boundary is propagated with the constant velocity $D = I_0/\varphi_*(r_0)$.

In conclusion the authors thank O. N. Krokhina for a discussion of the problems considered in the paper.

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